Math 103 Day 20: The Fundamental Theorem of Calculus and Indefinite Integrals

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Outline

Properties of Integrals

1. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
2. $\int_a^a f(x)dx = 0$
3. $\int_a^b c \;\; dx = c(b-a)$ where c is any constant.
4. $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
5. $\int_a^b cf(x)dx = c\int_a^b f(x)dx$ where c is a constant.
6. $\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$

Theorem

(Fundamental Theorem of Calculus, Part 1) If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)dt \quad a \leq x \leq b$$

is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

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Theorem

(Fundamental Theorem of Calculus, Part 2) If f is continuous on $[a,b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where F is any antiderivative of f, that is, a function such that $F' = f$.

Definition

$$\int f(x)dx = F(x) \;\; means \;\; F'(x) = f(x)$$

Exercise Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.